CONTRACTION OF A HIGHLY NONEQUILIBRIUM CURRENT-BEARING PLASMA

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"Contracted" is the term applied to that inhomogeneous state of a plasma in which it withdraws from the enclosing walls and concentrates in a more or less thin layer through which a current passes. Contraction is the result of instability developed in the original homogeneous state and may be related to the existence of a volt–ampere characteristic segment with negative differential conductivity. This phenomenon is known in semiconductor physics, and various instability mechanisms leading to contraction have been studied [1]. Well known in a low-temperature plasma is thermal contraction connected with superheating instability of the electron gas [2-4]. In the present study we will consider a highly nonequilibrium plasma in which contraction may develop as a result of disproportion in the number of electrons, i.e., contraction of a recombination–ionization character. We consider below the homogeneous state of a nonequilibrium weakly ionized plasma with charged-particle concentration $n_e \simeq 10^{11}$–10^{13} cm^{-3} (electron temperature T of the order of thousands of degrees, with gas cold). Disequilibrium is produced by the departure of radiation beyond the limits of the plasma volume. Such a state will be considered with respect to the instability noted, but not studied, in [5]. As a consequence of this instability the plasma may transform to an inhomogeneous (contracted) state, which is considered under conditions such that Joulean electron heating is compensated by losses due to elastic collisions with atoms of the gas. Charge diffusion plays the basic role in establishing the boundaries dividing the current-bearing region from that without current. More complex is the situation where radiation losses of energy are also significant and superheating, as well as ionization instability, is possible. This case is evaluated briefly at the close of the study.

1. The Homogeneous State of a Strongly Nonequilibrium Plasma. We will consider a weakly ionized homogeneous plasma in an electric field $\vec{\mathscr{E}}$, in which the electron temperature T significantly exceeds the atom and ion temperature T_a. The electrons are heated by a current and lose energy by elastic collisions with atoms, as well as radiation. The electron energy balance will be

$$\sigma\mathscr{E}^2 = W_{el} + W_R.$$

where $\sigma = n_e e^2/m\nu$ is the coefficient of electrical conductivity, $W_{el} = (2m/M)n_e\nu T$ are the losses in elastic collisions; N_e is the electron concentration; m and M are the masses of the electron and atom, respectively; ν is the elastic electron–atom collision frequency; W_R is the energy loss by radiation, mainly on atomic spectral lines. We will assume that collisions with ions are insignificant.

For sufficiently high n_e, its value will be determined by the Saha equation with temperature T. Such a plasma is referred to as "two-temperature." However, due to intense scintillation the population of atomic levels may become significantly less than the Boltzmann level, and n_e, less than the value given by the Saha equation. There then develops a non-two-temperature, highly nonequilibrium plasma [6]. We will consider its state.

To determine n_e and the populations of atomic levels n_k we write the particle balance equations for these levels. Relative scintillation intensity drops rapidly with growth in level number k. It is usually

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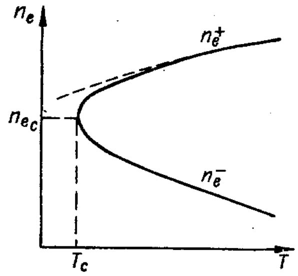

Fig. 1

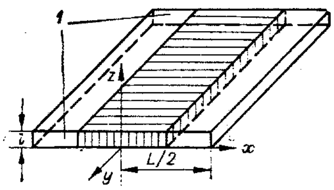

Fig. 2

sufficient to consider scintillations of only the first excited states k = 2, 3 (k = 1 being the ground state). Thus, having traversed the intervals 1 → 2, 2 → 3 a bound electron moves (diffuses) in energy space only as a result of collisions. We write the balance equation in the form of an expression for electron current j_ε in atomic level space [7],

$$j_\varepsilon = n_1 n_e \omega_{12} - n_2 n_e \omega_{21} - n_2 A_{21}; \quad \frac{\partial n_e}{\partial t} = j_\varepsilon, \tag{1.1}$$
$$j_\varepsilon = n_2 n_e \omega_{23} - n_3 n_e \omega_{32} - n_3 A_{32},$$
$$j_\varepsilon = n_3 n_e \omega_{3e} - n_e^3 \omega_{e3},$$

where n_1, n_2, n_3 are atomic concentrations; $n_e\omega_{k,k\pm1}$ are frequencies of inelastic collisions with electrons with transition $k \to k \pm 1$. We consider only transitions between neighboring levels, as they are the strongest $A_{k,k-1}$ is the probability of scintillation, considering possible resorption [6].

In reality, $\omega_{k,k+1}$ and $A_{k,k-1}$ are somewhat more complex quantities, defined in [7]. In that study the kinetics of shock radiation recombination and ionization are analyzed in greater detail, and some of the assumptions utilized herein are arrived at. It is shown that

$$\omega_{12} = \frac{4\sqrt{2\pi}e^4 \Lambda_1}{\sqrt{mT}(E_1 - E_2)} \exp\left(-\frac{E_1 - E_2}{T}\right), \tag{1.2}$$
$$\omega_{23} = \frac{4\sqrt{2\pi}\,\bar{\Lambda} e^4 E_1}{\sqrt{mT}(E_1 - E_3)(E_2 - E_3)} \exp\left(-\frac{E_2 - E_3}{T}\right),$$

where E_k is the energy of a level measured from the continuum (such that E_1 is the energy of ionization); Λ_1 and $\bar{\Lambda}$ are constants, $\omega_{k+1,k}$ and $\omega_{k,k+1}$, as well as ω_{e3} and ω_{3e},* are related by the equations

$$\omega_{k+1,k} = \omega_{k,k+1}\frac{K_{k+1}}{K_k}; \quad \omega_{3e} = \omega_{e3}K_3;$$
$$K_k = \frac{2\Sigma_i (2\pi m T)^{3/2}}{g_k h^3} \exp(-E_k)T,$$

Σ_i is the statistical ion sum, and g_k is the statistical weight of level K.

In the stable state scintillation is compensated by excitation and ionization and the value of $j_\varepsilon = 0$. From Eq. (1.1) we have

$$n_1 = \frac{n_e^2}{K_1}\left(1 + \frac{p_2}{n_e}\right)\left(1 + \frac{p_3}{n_e}\right); \quad n_2 = \frac{n_e^2}{K_2}\left(1 + \frac{p_3}{n_e}\right); \tag{1.3}$$
$$n_3 = n_e^2 K_3,$$

where $p_2 = A_{21}/\omega_{21}$, $p_3 = A_{32}/\omega_{32}$ are quantities characterizing scintillation. If $p_2, p_3 \gg n_e$, the stable plasma is in a state of strong disequilibrium. Such, for example, is the case with Ar + Cs plasma at T = 2000°K; $n_e = 10^{12}$ cm^{-3}; $n_{Cs} = 10^{15}$ cm^{-3}; $n_{Ar} = 10^{18}$ cm^{-3}.

It is significant that if n_e is small, interelectron collisions cannot maintain a Maxwell distribution at high energies. Thus, for ω_{12} we introduce the coefficient F_1 [7],

$$F_1 \simeq \frac{1}{1 + c}, \quad c = \frac{n_1\omega_{12}}{n_e\omega_{ee}} = \frac{2n_1}{n}\frac{T}{E_1 - E_2}\frac{\Lambda_1}{\lambda},$$

where λ is the "Coulomb" logarithm. Deviations from the Maxwell distribution greatly attenuate the direct ionization path, and we thus neglect it.

The total number of heavy particles $n = n_1 + n_e$ is fixed. If $n_e \ll n$, then $n \simeq n_1$, and for n_e we have from Eq. (1.3)

$$n_e^3 + n_e^2(p_2 + p_3) + n_e\left(p_2 p_3 + p_2\frac{\omega_{12}}{\omega_{ee}}n - nK_1\right) + p_2 p_3 \frac{\omega_{12}}{\omega_{el}}n = 0. \tag{1.4}$$

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* ω_{e3} is close in value to the recombination coefficient in the diffusion approximation of [8]:

$$\omega_{e3} \simeq 4\sqrt{2\pi}e^{10}\,\bar{\Lambda}\,(9\sqrt{m})^{-1}T^{-9/2}.$$

If $nK_1 > p_2p_3 + p_2(\omega_{12}/\omega_{ee})n$, then Eq. (1.4) has two positive roots, n_e^+ and n_e^- (Fig. 1). Consequently, for a given T, the existence of two homogeneous stationary states is possible: n_e^+ corresponds to a plasma relatively close to equilibrium and at higher values changes to the Saha formula $n_e^+ \simeq \sqrt{nK_1}$; n_e^- corresponds to a strong nonequilibrium plasma. When n_e^- is small, radiation output on both lines is significant, and the deviation from Maxwellian distribution will be

$$n_e^- \simeq \frac{1}{K_1} p_2p_3 \frac{\omega_{12}}{\omega_{ee}}. \tag{1.5}$$

At $T < T_c$ (Fig. 1) the homogeneous state cannot be realized.

2. Instability and Contraction of a Highly Nonequilibrium Plasma with Current. Elastic Losses. The thermal contraction of a current-carrying plasma normally studied occurs in the case of an inhomogeneous dependence of T on heating field $\mathscr{E}$ developing in the energy balance [4]. To a fixed value of $\mathscr{E}$, there correspond two homogeneous states with differing temperatures T. One of these is usually unstable due to disproportion between Joulean heating and electron energy losses. In an attempt to realize this state the plasma contracts [1, 4]. This mechanism will not be considered in this section. The energy balance equation with elastic losses W_{el}, into which enters the constant collision frequency ν, is stable with respect to superheating. It gives a single-valued relationship between T and $\mathscr{E}$

$$T = \frac{Me^2}{2m^2\nu^2} \mathscr{E}^2. \tag{2.1}$$

However, it was shown above that the function $n_e(T)$ is not single valued, and so to a field value $\mathscr{E}$ there correspond two homogeneous states with differing n_e. To determine the realizable homogeneous state it is necessary to study the solutions n_e^+, n_e^- with respect to stability relative to small fluctuations in electron density $\delta_e(t) = \delta_e e^{\gamma t}$ for a fixed field $\vec{\mathscr{E}}$. Linearizing system (1.1), we obtain

$$\delta j_e = -\frac{E_1}{T^2} \frac{n_e}{\tilde{n}_e} \frac{\delta_e}{\tau}; \quad \gamma = -\frac{E_1}{T^2} \frac{n_e}{\tilde{n}_e} \frac{1}{\tau}, \tag{2.2}$$

where $\tilde{n}_e = dn_e(T)/dT$ is the temperature derivative of the homogeneous background density (Fig. 1), and τ is the characteristic recombination time, while

$$\tau = \frac{K_1}{n_e^2\omega_{12}F_1(1 + p_2/n_e)(1 + p_3/n_e)} + \frac{K_2}{\omega_{23}(1 + p_3/n_e)n_e^2} + \frac{K_3}{\omega_{3e}n_e^2}.$$

It follows from Eq. (2.1) that the strongly nonequilibrium state at $n_e < n_{ec}$; $\tilde{n}_e < 0$ (Fig. 1) is unstable. In a strongly nonequilibrium plasma $\tilde{n}_e^- = -n_e^- E_1/T^2$ [Eq. (2.2)], and thus, the instability development time is close to τ. The plasma either enters the state with increased n_e, i.e., n_e^+, or decays. As was indicated above, the presence of unstable homogeneous states leads to the appearance of a falling segment in the volt–ampere characteristic $\vec{j}\vec{\mathscr{E}}$. In fact, the form of the function $\vec{j} = e^2 n_e(T) \times \vec{\mathscr{E}}/m\nu$, where T corresponds to Eq. (2.1), is close to that of $n_e(T)$ (Fig. 1), i.e., has an S-shaped character.

There are, however, possible conditions under which homogeneous fluctuations cannot develop. If because of high external circuit resistance a constant total current I is maintained, then only inhomogeneous fluctuations can develop [1]. In studying inhomogeneous fluctuations $\delta(\vec{r}t) = \delta_0 \exp(i\vec{k}\cdot\vec{r} + \gamma t)$ in system (1.1) we must consider ambipolar diffusion and supplement the system with the equations

$$\frac{\partial n_e}{\partial t} = j_\varepsilon(n_eT) + D_a\nabla^2 n_e, \tag{2.3}$$

$$\sigma(n_e)\mathscr{E}^2 - W_{el}(n_e, T) = E_1 j_\varepsilon(n_eT) - \nabla(\lambda_e \nabla T), \tag{2.4}$$

$$\vec{j} = \sigma(n_e)\vec{\mathscr{E}}, \ \operatorname{div}\vec{j} = 0, \ \operatorname{rot}\vec{\mathscr{E}} = 0. \tag{2.5}$$

Here $\lambda_e = n_eT/m\nu$ is the coefficient of electrode thermal conductivity. Linearizing Eqs. (1.1), (2.3)-(2.5) for small inhomogeneous perturbations δ_e, δ_T, $\delta\vec{\mathscr{E}}$, δj_ε and considering only exponential dependence on T, we obtain

$$\delta j_\varepsilon = -\frac{E_1}{T^2} \frac{n_e}{\tilde{n}_e} \frac{\delta_e}{\tau} + \frac{E_1}{T^2} \frac{n_e}{\tau} \delta_T, \tag{2.6}$$

$$\delta j_\varepsilon = \delta_e(\gamma + D_a k^2),$$

$$2\sigma\vec{\mathscr{E}}\delta\vec{\mathscr{E}}+\left(\mathscr{E}^2\frac{\partial\sigma}{\partial n_e}-\frac{\partial W_{el}}{\partial n_e}\right)\delta_e-E_1\delta j_\varepsilon-\left(\frac{\partial W_{el}}{\partial T}+k^2\lambda_e\right)\delta_T=0,$$

$$\delta\vec{\mathscr{E}}=-\frac{1}{\sigma}\frac{\partial\sigma}{\partial n_e}(\vec{\mathscr{E}}\cdot\vec{\mathbf{k}})\frac{\vec{\mathbf{k}}}{k^2}\delta_e.$$

The condition for nontriviality of system (2.6) gives a dispersion equation, which we write, directing the y axis along the field $\vec{\mathscr{E}}$

$$\gamma=-\frac{E_1}{T^2}\frac{n_e}{\tilde{n}_e}\frac{1}{\tau}\left[2\frac{T\tilde{n}_e}{\tau_{el}}\frac{k_y^2}{k^2}+\frac{n_e}{\tau_{el}}+\lambda_e k^2\right]\left|\frac{E_1^2}{T^2}\frac{n_e}{\tau}+\frac{n_e}{\tau_{el}}+\lambda_e k^2\right]^{-1}-D_a k^2, \qquad (2.7)$$

where $\tau_{el}=[2m\nu/M]^{-1}$ is the relaxation time T for elastic collisions. The maximum increment occurs for perturbations with $k_y=0$, leading to a layering of the plasma across the current. Only such perturbations will be considered further. Fluctuations with dimensions smaller than

$$L_D=\pi\left(\frac{T^2}{E_1}\frac{|\tilde{n}_e|}{n_e}\tau D_a\right)^{1/2}\simeq\pi(\tau D_a)^{1/2} \qquad (2.8)$$

damp out even if $\tilde{n}_e<0$. They are reabsorbed by diffusion in a time shorter than the ionization–recombination time τ. For $n_e^-\to n_{ec}$, $|\tilde{n}_e^-|\to\infty$ and the dimension L_D increases without limit. For $n_e\geq n_{ec}$ fluctuations of any dimension damp out.

If the system dimensions are greater than L_D and $n_e<n_{ec}$, then as a result of the development of instability, the plasma may transform to a stationary inhomogeneous state. We will consider the simple geometery proposed in [4], where the volume V has the form of a long thin plane layer (Fig. 2) with dimensions $l<L_D$, $L>L_D$. The resulting stationary inhomogeneous distributions of n_e and T will be described by the system of equations* (2.3), (2.4) with $(\partial n_e/\partial t)=0$. The solutions of interest to us have the form of a stable current pinch, separated from the currentless zone by a narrow front within which n_e and T are constant. From Eq. (2.7) it follows that thermal conductivity cannot form a stable front, since the instability produced by disproportion in particle balance is not stabilized. However, simultaneous solution of Eqs. (2.3), (2.4) is difficult, and so we will consider the limiting cases of very high and very low thermal conductivity below.

The dimensions of the front L_f are close to the critical fluctuation wavelength, whose wave vector k_c is given by the equation $\gamma=0$ [9]

$$\frac{E_1}{T^2}\frac{n_e}{\tilde{n}_e}\frac{1}{\tau}\left[\frac{n_e}{\tau_{el}}+\lambda_e k_c^2\right]\left[\frac{E_1^2}{T^2}\frac{n_e}{\tau}+\frac{n_e}{\tau_{el}}+\lambda_e k_c^2\right]^{-1}+D_a k_c^2=0. \qquad (2.9)$$

Its solution can be written. However, it is evident that if the inequality

$$\frac{n_e}{E_1|\tilde{n}_e|}\frac{\lambda_e}{n_e D_a}\simeq\frac{D_e}{D_a}\left(\frac{T}{E_1}\right)^2\gg 1 \qquad (2.10)$$

is fulfilled, where D_e is the electron diffusion coefficient, then the front width L_f is equal to L_D [Eq. (2.8)], and Eq. (2.10) takes on the form $\lambda_e k_c^2\gg(E_1^2/T^2)(n_e/\tau)$. Equation (2.8) for L_f may also be obtained while neglecting temperature fluctuations from the very onset. Thus, inequality (2.10) corresponds to a plasma with elastic losses in the volume and high thermal conductivity which levels the temperature. Inhomogeneity in T can develop in the front only because of the appearance in Eq. (2.4) of a loss $E_1 j_\varepsilon$. Its development time is $(T^2/E_1^2)\tau$, and it is levelled by thermal conductivity over a large distance of the order of

$$L_T\simeq\left(\frac{\lambda_e\tau}{n_e}\frac{T^2}{E_1^2}\right)^{1/2}\simeq\left(D_e\tau\frac{T^2}{E_1^2}\right)^{1/2}\gg L_D.$$

Consequently, the temperature T_0 in pinch and front are homogeneous, and from Eq. (2.1)

$$T_0=\frac{Me^2}{2m^2\nu^2}\mathscr{E}^2.$$

We will consider a solution contracted about the x axis (Fig. 2). It is described by the equation

$$D_a\frac{d^2 n_e}{dx^2}+j_\varepsilon(n_e,T_0)=0 \qquad (2.11)$$

* As in [4], we will neglect losses at the walls, considering only volume processes.

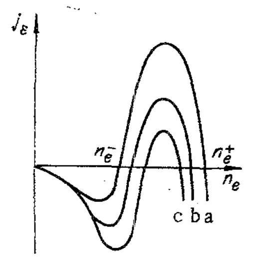

Fig. 3

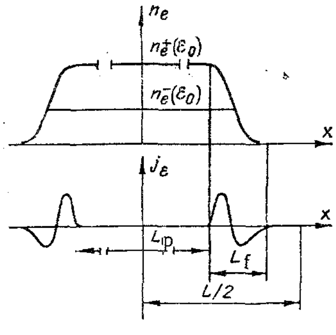

Fig. 4

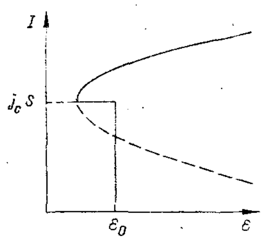

Fig. 5

for $y = \pm L/2$ (electron flow through the boundary is equal to zero). The function

$$j_\varepsilon = \frac{n_e}{\tau}\left[\frac{K_1 n}{n_e^2\,(1+p_2/n_e\cdot F_1)\,(1+p_3/n_e)} - 1\right] \tag{2.12}$$

follows from Eq. (1.1) and is shown in Fig. 3. The characteristic form of the solution studied (Fig. 4) is a homogeneous layer with $n_e > n_{e_c}$ surrounded by a region where n_e is close to zero. Using the phase trajectory method [9], it can be shown that such a solution exists for a temperature value satisfying the condition* $\int\limits_0^{n_e^+(T_0)} j_\varepsilon\,(n_e^1, T_0)\,dn_e^1 = 0.$

Through reduction in current, for example, by increasing external circuit resistance, one may attempt to realize the homogeneous state with current density $j < j_c$ (Fig. 5). Then at a current $I = j_c \cdot S$, where S is the area of the electrode surfaces, the plasma transforms to the contracted state with a current density in the current pinch $j_p = \sigma\,[n_e^+(\mathscr{E}_0)]\cdot\mathscr{E}_0$ and a field intensity $\mathscr{E}_0 = \sqrt{2T_0 m^2 v^2 (M^{-2})}$. The size of the pinch L_p is determined by the current $I = j_p \cdot S \cdot L_p/L$ and decreases with decrease in I. The width of the transition region is of the order of L_D. The remaining solutions of this type (for example, alternating layers) are unstable [9].

Calculations were performed for an Ar plasma with conditions $n = 10^{18}\ \mathrm{cm}^{-3}$, $T_a = 300°\mathrm{K}$, $l = 0.2$ cm. The effect of disequilibrium begins to appear at $n_e \lesssim 10^{14}\ \mathrm{cm}^{-3}$; the value of T_c (Fig. 1) is close to 7000°K. Under these conditions contraction corresponds to a field of $\mathscr{E}_0 \simeq 0.5$ V/cm.

We will now consider the opposite case of weak thermal conductivity, where

$$\frac{n_e}{E_1\,|\tilde{n}_e|}\,\frac{\lambda_e}{D_a n_e} \ll 1. \tag{2.13}$$

Then from Eq. (2.9) we arrive at the expression

$$L = \pi\sqrt{\frac{T^2}{E_1}\,\frac{|\tilde{n}_e|}{n_e}\,\tau D_a\left(1 + \frac{E_1^2}{T^2}\,\frac{\tau_{el}}{\tau}\right)}. \tag{2.14}$$

The condition of Eq. (2.13) itself takes on the form $\lambda_e k_c^2 \ll (E_1^2/T^2)(n_e/\tau)[1+(E_1^2/T^2)(\tau_{el}/\tau)]^{-1}$, where $(T^2/E_1^2)\tau\,[1+(E_1^2/T^2)(\tau_{el}/\tau)]$ is the development time of temperature inhomogeneity. Temperature fluctuations will be levelled by thermal conductivity if their dimensions do not exceed the small value

$$L_\lambda^1 = \pi\sqrt{\frac{T^2}{E_1}\,\frac{\tau\lambda_e}{n_e}\left(1 + \frac{E_1^2}{T^2}\,\frac{\tau_{el}}{\tau}\right)} \ll L_c.$$

Consequently, thermal conductivity does not affect temperature inhomogeneities of dimensions L_c and greater. The term $(d/dx)[\lambda_e(dT/dx)]$ in Eq. (2.4) will be neglected. The distribution of T over the front depends on the ratio of two types of losses – elastic and inelastic:

$$T = T_0\left(1 - \frac{E_1}{T_0}\,\frac{\tau_{el}}{n_e}\,j_\varepsilon\,(n_e, T)\right). \tag{2.15}$$

Considering the coarse estimate $|j_\varepsilon| \simeq n_e/\tau$, from Eqs. (2.14), (2.15) it may be concluded that if the ionization time is great, $E_1\tau_{el} \ll T_0\tau$, we then return to the previously considered isothermal case. Elastic en-

* The condition follows from Eq. (2.1) and accompanying boundary condition

$$\int\limits_0^{n_e^+(T_0)} j_\varepsilon\left(n_e^1 T_0\right) dn_e^1 = \frac{1}{2} D_a \times \left(\frac{dn_e}{dx}\right)^2 \Bigg|_{-L/2}^{+L/2} = 0.$$

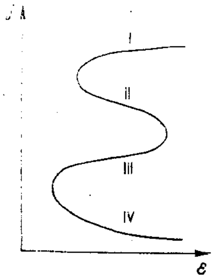

Fig. 6

ergy losses equalize weak inhomogeneities produced by ionization–recombination. On the other hand, if $E_1\tau_{el} \gg T_0\tau$, then the T distribution in the front may be quite inhomogeneous. In the latter case it is necessary to solve the system (2.3), (2.15), since the effect of temperature change on $j_{\mathcal{E}}(n_e, T)$ cannot be neglected. The field $\mathcal{E}_0$ at which contraction occurs is determined by the condition that

$$\int_0^{n_e^+(\mathcal{E}_0)} j_{\mathcal{E}}\left(n_e^1, T\left(n_e^1, \mathcal{E}_0\right)\right) dn_e^1 = 0, \tag{2.16}$$

where the function $T(n_e, \mathcal{E})$ is found from Eq. (2.15). For a coarse estimate of $\mathcal{E}_0$ we will proceed as follows. The inequality $E_1\tau_{el} \gg T_0\tau$ permits us to neglect elastic energy losses. Then for $j_{\mathcal{E}}$ we have the approximate expression $j_{\mathcal{E}}(n_e, T) \simeq (T_0/E_1)(n_e/\tau_{el})$. This estimate is, however, invalid in the vicinity of the zeroes of the function $j_{\mathcal{E}}$. The position of the latter is given by the expression in brackets in Eq. (2.12) at a temperature equal to T_0. Thus, over the entire parameter range $j_{\mathcal{E}}$ is described fairly well by the expression

$$j_{\mathcal{E}} \simeq \frac{T_0}{E_1}\frac{n_e}{\tau_{el}} f(n_e, T_0), \tag{2.17}$$

where $f(n_e, T_0)$ is a function which goes to zero at the points $n_e^+(T_0)$ and $n_e^-(T_0)$ and is equal to -1 at $n_e < n_e^-(T_0)$ and $+1$ at $n_e^-(T_0) < n_e < n_e^+(T_0)$. Using Eq. (2.17) in Eq. (2.16) greatly simplifies the problem. The condition of Eq. (2.16) then takes on the form

$$\int_0^{n_e^+(\mathcal{E}_0)} n_e^1 f\left(n_e^1, \mathcal{E}_0\right) dn_e^1 = 0,$$

whence follows the equation for estimation of $\mathcal{E}_0 : n_e^+(\mathcal{E}_0) = \sqrt{2}n_e^-(\mathcal{E}_0)$. The "renormalization" of $j_{\mathcal{E}}$ [Eq. (2.17)], obtained by consideration of temperature change, leads to a change in the time of development of ionization fluctuations $\tau \rightarrow (E_1^2/T_0^2)\tau_{el}$. This affects the front dimensions

$$L_c = \pi\left(\frac{E_1|\tilde{n}_e|}{n_e}\tau_{el}D_a\right)^{1/2} \simeq \left(\frac{E_1^2}{T_0^2}\tau_{el}\cdot D_a\right)^{1/2}\cdot\pi,$$

which corresponds to Eq. (2.14) at $(E_1^2/T^2)(\tau_{el}/\tau) \gg 1$.

3. Contraction of a Plasma with Current. Elastic and Radiation Losses. It is known that radiation energy losses W_e lead to superheating instability [4]. Thus, a nonequilibrium plasma with current with consideration of elastic and radiation losses will be subjected to two types of instability – ionization and superheating.

According to the general considerations of [9], instability of a homogeneous current-bearing plasma will appear if there exists a segment of the volt–ampere characteristic with negative differential conductivity. Such a characteristic is shown in Fig. 6. Segment I corresponds to strong ionization; the function $n_e(T)$ is given by the Saha formula; elastic losses predominate in the energy balance equation. On segment II the electron density is still high, and the effect of nonequilibrium is insignificant. Radiation losses predominate in the energy balance, which produces radiation superheating [4]. On segment III radiation losses are also significant but the plasma state here is in great disequilibrium, which leads to stabilization. A plasma containing 10^{15} cm^{-3} atoms of Cs and 10^{18} cm^{-3} atoms of Ar at T = 2000°K is in such a state. The electron density is approximately 10^{12} cm^{-3}. Finally, on segment IV radiation losses become small (n_k are small), i.e., a strongly nonequilibrium plasma with losses W_{el} develops, which is ionization unstable.

We now write the damping decrement of small perturbations for the case where $W_R \gg W_{el}$ and the major contribution to W_R comes from transitions $2 \rightarrow 1$ $W_R = (E_1 - E_2)n_2A_{21}^*$ (which is intrinsic to alkaline metal plasmas):

$$\gamma = -\frac{E_1}{T^2}\frac{n_e}{\tilde{n}_e}\frac{1}{\tau}\left[n_e\frac{d}{dT}\left(\frac{W_R}{n_e}\right) + 2\frac{k_y^2}{k^2}\frac{W_R}{n_e}\tilde{n}_e + k^2\lambda_e\right]\times$$
$$\times\left[\frac{E_1^2}{T^2}\frac{n_e}{\tau} + \frac{(E_1 - E_2)^2}{T^2}\frac{n_e}{\tau_2^R}\left(1 - \frac{K_1}{\omega_{12}n_e^2\tau}\cdot\frac{E_1}{E_1 - E_2}\times\right.\right.$$
$$\left.\left.\times\frac{1}{F_1(1 + p_2/n_e)(1 + p_3/n_e)}\right) + k^2\lambda_e\right]^{-1} - D_ak^2$$

where $\tau_2^R = n_e/n_2A_{21}^*$ is the time characterizing second-level scintillation. A characteristic of superheating instability is the negative sign of the derivative $d/dT(W/\sigma)$ [9]. Along the current ($k_y = k$) superheating instability will be stabilized by the term $2(\tilde{n}_e/n_e)W_R(k_y^2/k^2)$; therefore, we will consider only transverse fluctuations with $k_y = 0$.

On segment II, where we have a two-temperature plasma,

$$\gamma = \frac{E_1}{T^2}\frac{n_e}{|\tilde{n}_e|}\frac{1}{\tau}\left[\frac{n_e}{\tau_2^R}\frac{(2E_2-E_1)(E_1-E_2)}{2T^2} - k^2\lambda_e\right]\times\left[\frac{E_1^2}{T^2}\frac{n_e}{\tau} + \frac{n_e}{\tau_2^R}\frac{(E_1-E_2)^2}{T^2}\left(1-\frac{K_1}{\omega_{12}\tau n_e^2}\cdot\frac{E_1}{E_1-E_2}\right) + k^2\lambda_e\right]^{-1} - D_a k^2. \quad (3.1)$$

The front of the contracted state developing at $2E_2 > E_1$ in the case of superheating [4] may be formed by electron thermal conductivity and has a value of the order of

$$L_\lambda = \pi\left[\frac{\tau_2^R\lambda_e}{n_e}\frac{2T^2}{(2E_2-E_1)(E_1-E_2)}\right]^{1/2}.$$

However, from Eq. (3.1) it is evident that if $L_\lambda \ll L_D$, the front is formed by diffusion. Its width is equal to

$$L_c = \pi\left[\frac{T^2}{E_1}\frac{|\tilde{n}_e|}{n_e}D_a\tau\frac{2(E_1-E_2)}{2E_2-E_1}\left(1+\frac{E_1^2}{(E_1-E_2)^2}\frac{\tau_2^R}{\tau}\right)\right]^{1/2}$$

and at $\tau \gg \tau_2^R$† is close to L_D.

On segment III, where $\tilde{n}_e < 0$ and $d/dT(W_R/\sigma) < 0$, for γ we obtain

$$\gamma = \frac{n_e}{|\tilde{n}_e|E_1}\frac{E_1^2}{T^2}\frac{1}{\tau}\left[k^2\lambda_e - \frac{(E_1-E_2)E_2}{T^2}\frac{n_e}{\tau_2^R}\right]\left[\frac{E_1^2}{T^2}\frac{n_e}{\tau} + \frac{(E_1-E_2)^2}{T^2}\frac{n_e}{\tau_2^R} + k^2\lambda_e\right]^{-1} - D_a k^2.$$

Despite the presence of attributes of both ionization and superheating instabilities, fluctuations decay if

$$\frac{\lambda_e}{D_a n_e}\frac{n_e}{|\tilde{n}_e|E_1} < 1 + \frac{\tau}{\tau_2^R}(2\beta-\beta^2) + \sqrt{\left(\frac{\tau}{\tau_2^R}\right)^2 2\beta^2(1-\beta)(2-\beta) + \frac{\tau}{\tau_2^R}2\beta(1-\beta)},$$

where $\beta = 1 - E_2/E_1$. But if this condition is not fulfilled there exist two values of wave number $k_{c_1} < k_{c_2}$ at which $\gamma = 0$. Fluctuations with $k_{c_1} < k < k_{c_2}$ in the linear approximation prove to be nondamping. In the limiting case of high thermal conductivity, where $\lambda_e[E_1|\tilde{n}_e|D_a n_e]^{-1} \gg 1$; $L_\lambda \gg L_D$, the characteristic dimensions corresponding to k_{c_1} and k_{c_2} have the form $L_{c_1} \simeq L_\lambda \gg L_{c_2} \simeq L_D$. Fine-scale perturbations of dimensions less than L_D are reabsorbed by diffusion; electrons succeed in leaving the fluctuation volume. Perturbations of dimensions $L_D < L < L_\lambda$ will grow, since diffusion cannot level the inhomogeneity in n_e, and the change in T will be compensated by electron thermal conductivity, so that the inverse effect of T on current j_ε will not exist. This factor comes into play only when the dimensions of the fluctuation (due to its spreading) exceed L_λ. Within the fluctuation there develops a change in T which leads to damping of the perturbation.

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† The condition of neglecting radiation in the kinetics $K_1/\omega_{12}n_e^2 \ll \tau_2^R$ does not contradict the condition $\tau \gg \tau_2^R$ if the "narrow spot" is located above the interval 1-2 and $\tau \gg K_1/\omega_{12}n_e^2$, which is valid for alkaline metals.

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